Letters

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References


Geometrical optics of the retinal image stabilisation device

Sir: An optical device giving partial or almost-complete stabilisation of the retinal image has been described and tested to treat patients with continuous oscillations caused by nystagmus, usually due to acquired neurological disease such as multiple sclerosis. It can relieve such oscillations, and consists of a high-plus spectacle lens used together with a high-minus contact lens.

In many of these patients the amplitude of oscillations is less than would be predicted from the amplitude of their nystagmus, so that the image only needs to be partially stabilised. A means is then needed for predicting how much retinal image stabilisation (“RIS”) will be needed in each case, and for calculating the correct lens power required to achieve it.

An index of oscillation has been developed, defined as the ratio of angular amplitude of oscillation (“O”) and angular amplitude of nystagmus (“N”). O/N can vary from 0 (no oscillation, in spite of nystagmus) to 1 (oscillation of the same amplitude as nystagmus). The index of retinal image stabilisation required is also equal to O/N, since the proportion of the nystagmus that is perceived as oscillation is also the proportion of the nystagmus that must be cancelled by the device.

Partially stabilising the retinal image is equivalent to increasing the amount of rotation of the eye required to move the image through a given angle on the retina (rotational magnification). Retinal image stabilisation therefore bears a simple relation with rotational magnification, which is a well-studied property of plus spectacles.

Rotational magnification of the image on the retina (RMR) is here defined as the ratio of the angular motion of the eye scanning a scene (N) and the resulting angle of retinal image-slip (P). RMR differs from the amount of rotational magnification in terms of distance across the scene (RMs) when, as here, the device causing the rotational magnification also causes angular magnification of the retinal image. What we need is to apply sufficient RMR to reduce the retinal image-slip (originally equal to N) by the amount of the oscillation (O). The remaining image-slip P does not cause oscillation since it is compensated by a central mechanism (“C”) whose size and mode of action is not known.

Then: $N = \text{amplitude of nystagmus}$

$O = \text{amplitude of oscillation}$

$C = \text{amplitude of central compensating signal}$

$P = \text{amplitude of residual retinal image-slip}$

$N = C + O (1)$

$RMR = N/P (2)$

From (1), $O/N = 1 - (C/N)$

From (2), $(RMR - 1)/RMR = 1 - (P/N)$

For optical neutralisation of oscillations, we have said that $P = C$

So $N/O = (RMR - 1)/RMR = RIS (3)$

The RMR is generated by the combination of spectacle and contact lens, which if the image is in focus on the retina form a galilean pair. A galilean pair is afoal, so where:

$fs = \text{focal length of spectacle (in metres)}$

$Ps = \text{power of spectacle (in dioptries)}$

$fc = \text{focal length of contact lens (in metres)}$

$Pc = \text{power of contact lens (in dioptries)}$

$d = \text{separation between principal planes of spectacle and contact lens (in metres)}$

$fs = -fc + d$

Or, $1/Ps = d - (1/Pc) (4)$

The galilean pair also causes angular magnification of the image (AM), which is given by:

$AM = -Ps/(Pc+Ps) (5)$

(see ref 4, p205).
Now we need to find how the amount of RIS obtained varies with the lens power.

The rotational magnification generated in terms of movement of the gaze across the scene (RMs) by a plus spectacle lens can be shown to be given by:

\[ RMs = \frac{1}{1 - Ps(d + r)} \] (6) (see ref 4, p248)

where: \( r \) = radius of the eye, from its centre of rotation to the principal plane of the contact lens
Ps = power of spectacle lens (in dioptres)

But what we need is the rotational magnification in terms of retinal image-slip (RMr), not in terms of movement of the gaze across the scene (RMs), because the scene is magnified by the optical device.

So, \( RMr = \frac{RMs}{AM} \).

Substituting into (6) to obtain RMr:

\[ RMr = \frac{1}{1 - Ps(d + r)]AM} \] (7)

Put (7) into (3):

\[ RIS = \frac{1}{1 - (d + r)Ps]AM} - 1 \]
\[ = \frac{1}{(1 - (d + r)Ps]AM} \]
\[ = 1 - AM[1 - (d + r)Ps] \] (8)

From (5), substitute into (8) for AM:

\[ RIS = 1 + \frac{Pc}{Ps} - \frac{Pc(d + r)}{Ps} \]

From (4), substitute for \( 1/Ps \):

\[ RIS = 1 + \frac{Pc(d - 1/Pc)}{Ps} - \frac{Pc(d + r)}{Ps} \]
\[ = 1 + \frac{d.Pc}{Ps} - 1 - d.Pc - \frac{Pc.r}{Ps} \]
\[ = \frac{Pc.r}{Ps} \]

So the amount of RIS obtained is directly proportional to the contact lens power and the radius of the eye. For each combination of \( Pc \) and \( d \), \( Ps \) is fixed by the need to form an afocal galilean pair. If \( d \) is varied, \( Ps \) and AM will vary as a consequence, but RIS will be unchanged. It follows that if \( r \) is taken to be a constant, a nomogram can be calculated giving all combinations of RIS, \( Ps \), \( Pc \) and \( d \).

An example of a nomogram for \( r = 1.25 \text{ cm} \) has been constructed. The results are only approximate, because the equations used throughout are those of thin-lens geometrical optics, while to obtain clinically appropriate amounts of RIS, \( Ps \) and \( Pc \) are both high.

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